```
Clear["Global`*"]
```

3 - 9 Steepest descent
Do steepest descent steps when:

3. $f[x] = 2 x_1{}^2 + x_2{}^2 - 4 x_1 + 4 x_2$, $x_0 = 0$, 3 steps

I replace the subscripted forms in the problem expression with x and y, which is easier for me to see. I need to mention the Wolfram demonstration by Michael Waters, *https://demonstrations.wolfram.com/CurvesOfSteepestDescentFor3DFunctions/*, which is appealing and gives the right answer. It is also accommodating about accepting an alien function into its central module.

```
Clear["Global`*"]
```

```
f[x_, y_] = 2 x^2 + y^2 - 4 x + 4 y
```

$-4 x + 2 x^2 + 4 y + y^2$

A plot may be useful in reconnoitering the district.

```
Plot3D[f[x, y], {x, 0, 15}, {y, 0, 15}, AxesLabel → Automatic,
  ImageSize → 150, AspectRatio → Full, ViewPoint -> {-4000, 2000, 1000}]
```
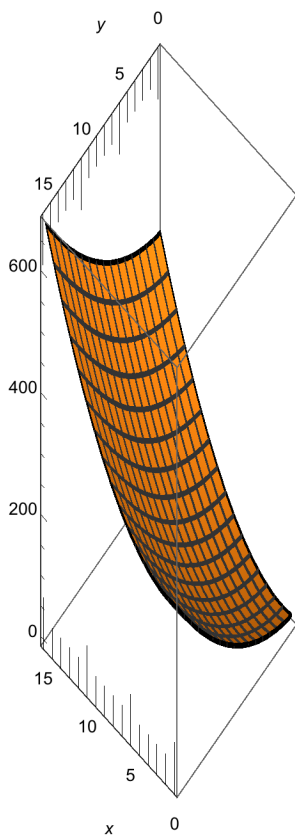
First objective is to establish the answer.

```
FindMinimum[f[x, y], {x, y}]
```

```
{-6., {x → 1., y → -2.}}
```

I see below that step 3 by Mathematica is very close to step 3 by the text answer procedure, whatever that may entail. The text answer for step 3 is {1.037, -1.926}.

```
pts = Reap[FindMinimum[f[x, y],
      {{x, -1.2}, {y, 1}}, StepMonitor ⧴ Sow[{x, y}]]][[2, 1]]
pts = Join[{{-1.2, 1}}, pts];
```

```
{{-0.554696, 0.56002}, {1.12183, -1.64264}, {1.04891, -1.92555},
 {1.00106, -2.00055}, {1.00003, -2.00003}, {1., -2.}, {1., -2.}}
```

```
ContourPlot[f[x, y], {x, -3, 3}, {y, -3, 3},
 Epilog → {Red, Line[pts], Point[pts]}, ImageSize → 200]
```
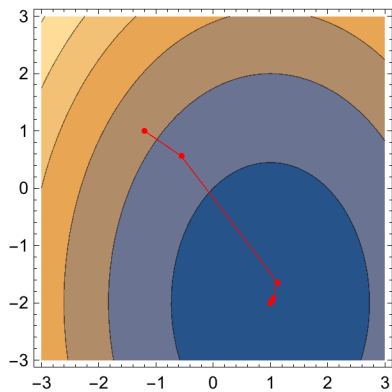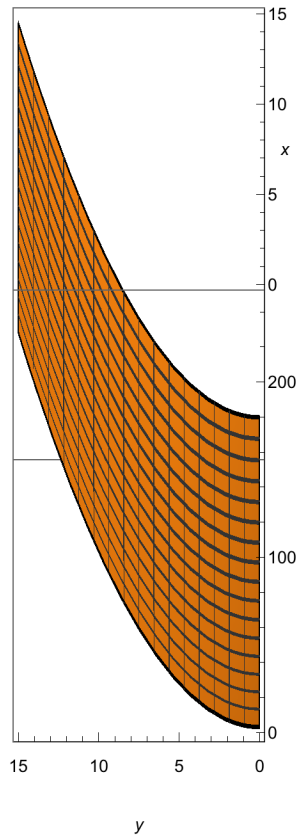

> 9. $f[x] = 0.1 x_1{}^2 + x_2{}^2 - 0.02 x_1$, $x_0 = (3, 3)$, 5 steps

```
Clear["Global`*"]
```

```
f[x_, y_] = 0.1 x^2 + y^2 - 0.02 x
```

$$-0.02 x + 0.1 x^2 + y^2$$

```
Plot3D[f[x, y], {x, 0, 15}, {y, 0, 15}, AxesLabel → Automatic,
 ImageSize → 150, AspectRatio → Full, ViewPoint -> {-4000, 0, 1000}]
```



```
FindMinimum[f[x, y], {{x, 3}, {y, 3}}]
```

$\left\{-0.001, \left\{x \to 0.1, y \to 5.96629 \times 10^{-11}\right\}\right\}$

The plot below makes me wonder if it is better not to give a guess about the preferred starting position. The text is interested in step 5, which in its search produced the coordinate {0.11247, -0.00012}.

```
pts =
 Reap[FindMinimum[f[x, y], {{x, 3}, {y, 3}}, StepMonitor ⧴ Sow[{x, y}]]][[
  2, 1]]
pts = Join[{{-1.2, 1}}, pts];
```

$\left\{\{2.79589, 0.888522\}, \{2.44241, -0.0226433\},\right.$
$\{2.16341, -0.0800494\}, \{0.368991, -0.13073\}, \{0.111153, -0.0394899\},$
$\{0.0942656, -0.0016205\}, \{0.0995506, -0.0000200394\},$
$\left\{0.0999956, 1.46532 \times 10^{-6}\right\}, \left\{0.1, 2.66175 \times 10^{-8}\right\}, \left\{0.1, 5.96629 \times 10^{-11}\right\}\right\}$

```
ContourPlot[f[x, y], {x, -3, 3}, {y, -3, 3},
 Epilog → {Red, Line[pts], Point[pts]}, ImageSize → 200]
```